8th Grade Math Packet

(For students entering 8th grade in August 2017)

This packet is OPTIONAL. If you complete it, you will earn BONUS HOMEWORK points.

You will get 2 bonus points for each section you complete up to a *maximum* of 50 points. In order to receive points for a section, you must:

- Show reasonable, clear work on every problem.
- Turn in your packet by September 1st, 2017.

Each section has a *worked example* and then some problems for you to try. If you are still feeling confused, type the section title into any of these websites:

khanacademy.org
ixl.com
Learnzillion.com

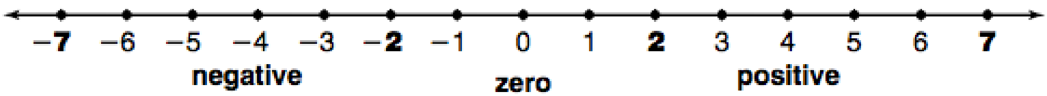
You may also email questions to azern@nhcsb.org

We will *not* be going over the answers to the packets when school begins, but there are answer keys at the school if you want to come check your answers.

You may not have a pencil or pen with you when you check your answers =)

Integers

The numbers 2 and −2 are opposites. The numbers 7 and −7 are opposites.
Integers are the set of positive whole numbers, their opposites, and zero.



You can use the number line to compare integers.

−2 is less than 0. $-2 < 0$	7 is greater than 2. $7 > 2$

Numbers to the left are less. −2 is farther left than 0.	Numbers to the right are greater. 7 is farther to the right than 2.

Compare using <, >, or =.

1. 4 ☐ 2 **2.** −3 ☐ −2 **3.** 3 ☐ −4

4. −1 ☐ −2 **5.** 0 ☐ 5 **6.** 0 ☐ −4

Find the opposite of each number.

13. 8 ________ **14.** −5 ________ **15.** 147 ________

Order the numbers from least to greatest.

22. −4, 5, −2, 0, 1 **23.** 6, −3, −5, 4, −6

Adding and Subtracting Integers

Use these rules to add and subtract integers.

Adding Integers

Same Sign	Different Signs
• The sum of two positive integers is positive. Example: $6 + 16 = 22$ • The sum of two negative integers is negative. Example: $-9 + (-3) = -12$	• First find the absolute values of each number. • Then subtract the lesser absolute value from the greater. • The sum has the sign of the integer with the greater absolute value. Example: $-10 + 9 = -1$

Subtracting Integers

- To subtract integers, add the opposite.
- Then following the rules for adding integers.

  Example: $6 - (-3) = 6 + 3 = 9$

Find each sum.

1. $8 + (-2)$ ______ **2.** $-9 + 4$ ______ **3.** $3 + (-2)$ ______

4. $-1 + 11$ ______ **5.** $12 + 13$ ______ **6.** $-9 + 5$ ______

7. $7 + 2$ ______ **8.** $-1 + (-7)$ ______ **9.** $-3 + 0$ ______

10. $-1 + (-1)$ ______ **11.** $6 + 5$ ______ **12.** $3 - (-2)$ ______

Complete.

13. $-3 - 4$ Change to addition: $-3 +$ ______ = ______

14. $5 - 2$ Change to addition: $5 +$ ______ = ______

15. $-6 - (-10)$ Change to addition: $-6 +$ ______ = ______

Find each difference.

16. $4 - 5$ ______ **17.** $-5 - 4$ ______ **18.** $-8 - (-7)$ ______

19. $19 - (-6)$ ______ **20.** $-10 - 12$ ______ **21.** $-12 - 10$ ______

Adding and Subtracting Decimals and Fractions

Use these rules to add and subtract rational numbers.

Adding and Subtracting Integers

Same Sign

- The sum of two positive rational numbers is positive.
  Example: $15.6 + 4.5 = 20.1$
  Example: $\frac{2}{9} + \frac{8}{9} = 1\frac{1}{9}$
- The sum of two negative rational numbers is negative.
  Example: $-3.42 + (-5.74) = -9.16$
  Example: $-1\frac{3}{4} + \left(-4\frac{3}{4}\right) = -6\frac{1}{2}$

Different Signs

- First find the absolute values of each addend.
- Then subtract the lesser absolute value from the greater.
- The sum has the sign of the addend with the greater absolute value.
  Example: $-25.8 + 17.3 = -8.5$
  Example: $-2\frac{1}{2} + 1\frac{1}{4} = -1\frac{1}{4}$

Subtracting Rational Numbers

- To subtract rational numbers, add the opposite.
- Then following the rules for adding rational numbers.
  Example: $-9.25 - (-3.4) = -9.25 + 3.4 = -5.85$
  Example: $4 - \left(-2\frac{3}{10}\right) = 4 + \left(2\frac{3}{10}\right) = 6\frac{3}{10}$

Find each sum.

1. $43.2 + 26.7$ __________

2. $-81.22 + 14.9$ __________

3. $-4.8 + (-53.5)$ __________

4. $2\frac{5}{9} + 3\frac{4}{9}$ __________

5. $-2\frac{3}{5} + 1\frac{1}{5}$ __________

6. $-6\frac{1}{3} + \left(-7\frac{1}{3}\right)$ __________

Find each difference.

7. $15.64 - 8.5$ __________

8. $-0.392 - 0.26$ __________

9. $-5.4 - (-1.6)$ __________

10. $6 - 5\frac{5}{6}$ __________

11. $-4\frac{3}{4} - 2\frac{1}{4}$ __________

12. $-7\frac{4}{5} - \left(-3\frac{3}{5}\right)$ __________

Multiplying and Dividing Integers

To multiply integers:

- If the signs are alike, the product is positive.

$$2 \cdot 3 = 6$$
$$-2 \cdot -3 = 6$$

- If the signs are different, the product is negative.

$$2 \cdot -3 = -6$$
$$-2 \cdot 3 = -6$$

To divide integers:

- If the signs are alike, the quotient is positive.

$$6 \div 3 = 2$$
$$-6 \div -3 = 2$$

- If the signs are different, the quotient is negative.

$$6 \div -3 = -2$$
$$-6 \div 3 = -2$$

Tell whether each product or quotient will be *positive* or *negative*.

7. $4 \cdot 7$ ____________

8. $-4 \cdot 7$ ____________

9. $-4 \cdot -7$ ____________

10. $4 \cdot -7$ ____________

11. $10 \cdot -4$ ____________

12. $-25 \div 5$ ____________

13. $-2 \cdot -2$ ____________

14. $100 \div 10$ ____________

Multiplying Decimals and Fractions

Remember these rules when multiplying rational numbers.

1. When both factors are positive, the product is positive.

Multiply: $\left(2\frac{2}{3}\right)\left(1\frac{5}{8}\right) = \left(\frac{8}{3}\right)\left(\frac{13}{8}\right) = \frac{104}{24} = 4\frac{1}{3}$

2. When both factors are negative, the product is positive.

Multiply: $(-4.35)(-2.44) = 10.614$

3. When both factors have different signs, the product is negative.

Multiply: $-\frac{3}{4} \times \frac{2}{5} = \frac{-3 \times 2}{4 \times 5} = \frac{-6}{20} = -\frac{6}{20} = -\frac{3}{10}$

Find each product. Write the product in simplest form.

1. 2.8×0.05 ____________

2. $\frac{5}{8} \cdot \frac{2}{5}$ ____________

3. $1.45 \cdot 0.7$ ____________

4. $2\frac{3}{5} \cdot \frac{7}{8}$ ____________

5. $(-2.07)(-4.9)$ ____________

6. $\frac{5}{12} \cdot \left(-\frac{3}{10}\right)$ ____________

Dividing Decimals and Fractions

Divide: $38.25 \div 1.5$.		Divide: $3\frac{3}{4} \div 1\frac{2}{5}$.	
1. Rewrite the problem with a whole number divisor.	$1.5\overline{)38.25}$	**1.** Rewrite mixed numbers as improper fractions as needed.	$\frac{15}{4} \div \frac{7}{5}$
2. Place the decimal point in the quotient.	$15\overline{)382.5}$ **Move 1 place each.**	**2.** Multiply by the reciprocal of the divisor.	$\frac{15}{4} \cdot \frac{5}{7}$
3. Divide. Then check.	$15\overline{)382.5}$ = 25.5; -30, 82, -75, 75, -75, 0 $25.5 \times 15 = 382.5$ ✔ **Multiply to check.**	**3.** Multiply numerators. Multiply denominators.	$\frac{15 \cdot 5}{4 \cdot 7} = \frac{75}{28}$
		4. Simplify.	$\frac{75}{28} = 2\frac{19}{28}$

Find each quotient. Simplify your answers.

1. $1\frac{5}{8} \div \frac{5}{8}$ ____________

2. $-43.55 \div 6.5$ ____________

3. $-\frac{2}{5} \div \frac{4}{25}$ ____________

Changing between Fractions and Decimals

To change a fraction to a decimal, divide the numerator by the denominator.

$\frac{3}{5}$ Think: $3 \div 5$

$$\begin{array}{r} 0.6 \\ 5\overline{)3.0} \\ \underline{-30} \\ 0 \end{array}$$

$\frac{3}{5} = 0.6$

To change a decimal to a fraction:

1. Read the decimal to find the denominator. Write the decimal digits over 10, 100, or 1,000.
2. 0.65 is 65 *hundredths* $\rightarrow \frac{65}{100}$

   Use the GCF to write the fraction in simplest form.

   The GCF of 65 and 100 is 5.

   $\frac{65}{100} = \frac{65 \div 5}{100 \div 5} = \frac{13}{20}$

Write each fraction as a decimal.

1. $\frac{4}{5}$ = ______ **2.** $\frac{3}{4}$ = ______ **3.** $\frac{1}{6}$ = ______

Write each decimal as a mixed number or fraction in simplest form.

10. 0.4 = ______ **11.** 0.75 = ______ **12.** 1.5 = ______

Evaluating and Writing Expressions

To evaluate an *expression*, substitute a value for the *variable* and compute.

Evaluate $5y - 8$ for $y = 7$.

$5y - 8$

$5 \times 7 - 8$ ← **Substitute y with 7.**

$35 - 8 = 27$ ← **Compute.**

You can use key words to write a word phrase for an algebraic expression.

$a + 5$	→	a plus 5
	or	a increased by 5
$2n$	→	the product of 2 and n
	or	2 times n

Evaluate each expression using the values $y = 4$, $z = 8$, and $p = 10$.

5. $3y + 6$ = ______ **6.** $4z - 2$ = ______

7. $p + 2p$ = ______ **8.** $3z \times z$ = ______

Write an algebraic expression for each word phrase.

13. x newspapers plus 10

____

14. 4 less than x teabags

____

15. 3 more than x envelopes

____

16. 6 times x school buses

____

Simplifying Expressions

To *simplify* an expression, combine its like terms.

Perform as many of its operations as possible.

Simplify: $6m + 10 - 2m + 4$

$= (6m - 2m) + (10 + 4)$

$= 4m + 14$

Simplify: $3(c - 6)$

$= 3c - 3(6)$

$= 3c - 18$

Simplify each expression.

1. $7(6 + p) =$ ________________ **2.** $6n + 2(4n + 5) =$ ________________

3. $3(0.3x + 0.1) + 0.2x =$ ________________ **4.** $-6x - 8 + 3x - 14 =$ ________________

Solving One-Step Equations

Follow these steps to solve equations by adding and subtracting.

	Solve: $n + (-2) = 11$	Solve: $n - 6 = -36$
① Use the inverse operation on both sides of the equation.	$n + (-2) - (-2) = 11 - (-2)$	$n - 6 + 6 = -36 + 6$
② Simplify.	$n = 13$	$n = -30$
③ Check.	$n + (-2) = 11$ $13 + (-2) \stackrel{?}{=} 11$ $11 = 11$ ✔	$n - 6 = -36$ $-30 - 6 \stackrel{?}{=} -36$ $-36 = -36$ ✔

Follow these steps to solve equations by multiplying and dividing.

	Solve: $\frac{t}{5} = -7$	Solve: $-2x = 8$
① Use the inverse operation on both sides of the equation.	$(5)\frac{t}{5} = (5)(-7)$	$\frac{-2x}{-2} = \frac{8}{-2}$
② Simplify.	$t = -35$	$x = -4$
③ Check.	$\frac{t}{5} = -7$ $\frac{-35}{5} \stackrel{?}{=} -7$ $-7 = -7$ ✔	$-2x = 8$ $-2(-4) \stackrel{?}{=} 8$ $8 = 8$ ✔

Use a calculator, pencil and paper, or mental math. Solve each equation.

5. $n + 1 = 17$ ______

6. $n - (-6) = 7$ ______

7. $n - 8 = -12$ ______

8. $\frac{x}{4} = -1$ ______

9. $-5w = 125$ ______

10. $\frac{m}{-8} = 10$ ______

Writing Equations for Word Problems

You can change a word expression into an algebraic expression by converting the words to variables, numbers, and operation symbols.

To write a two-step algebraic expression for *seven more than three times a number,* follow these steps.

① Define the variable.	Let n represent the number.
② Ask yourself if there are any key words.	"More than" means add and "times" means multiply.
③ Write an algebraic expression.	$7 + 3 \cdot n$
④ Simplify.	$7 + 3n$

Solve.

4. Three friends pay \$4 per hour to rent a paddleboat plus \$5 for snacks. Write an expression for the total cost of rental and snacks. Then evaluate the expression for 2 hours.

____

5. A lawn care service charges \$10 plus \$15 per hour to mow and fertilize lawns. Write an expression for the total cost of having your lawn mowed and fertilized. Then evaluate the expression for 4 hours.

____

Solving Two-Step Equations

Step 1

Subtract the extra marbles from both sides.

$$2x + 3 - 3 = 7 - 3$$
$$2x = 4$$

Step 2

Divide the number of marbles by 2, the number of boxes.

$$\frac{2x}{2} = \frac{4}{2}$$
$$x = 2$$

Complete to solve each equation.

3. $5x + 7 = 2$

$5x + 7 -$ ______ $= 2 -$ ______

$\frac{5x}{\square} = \frac{-5}{\square}$

$x =$ ______

4. $2x - 1 = 9$

$2x - 1 +$ ______ $= 9 +$ ______

$\frac{2x}{\square} = \frac{10}{\square}$

$x =$ ______

Solve each equation.

5. $4x + 7 = 15$ ______

6. $3b - 5 = 13$ ______

7. $5t - 2 = -17$ ______

Distributive Property

You can use the Distributive Property to solve equations.

Solve $4(x - 7) = 12$.

$4(x) + 4(-7) = 12$ ← **Use the Distributive Property.**
$4x - 28 = 12$ ← **Simplify.**
$4x = 40$ ← **Add 28 to each side.**
$x = 10$ ← **Divide each side by 4.**

Check $4(10) + 4(-7) = 40 + (-28)$

$4(10) + 4(-7) = 12$

Solve these equations.

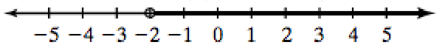
1. $-4(3.1 + x) = -38$ ______

2. $6(x - 7) = 30$ ______

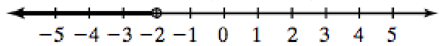
Inequalities

The **solutions of an inequality** are the values that make the inequality true. They can be graphed on a number line. Use a closed circle (●) for ≤ and ≥ and an open circle (**o**) for > and <. For example:

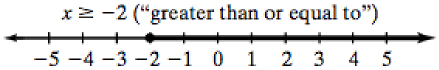
$x > -2$ ("greater than")



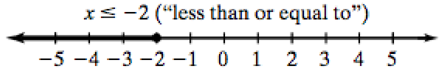
$x < -2$ ("less than")



$x \geq -2$ ("greater than or equal to")



$x \leq -2$ ("less than or equal to")



Graph the inequality $x > 4$.

The inequality is read as "x is greater than 4." Since all numbers to the right of 4 are greater than 4, you can draw an arrow from 4 to the right. Since 4 is not greater than itself, use an open circle on 4.

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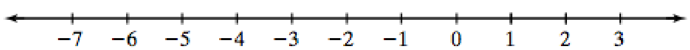
1. **Graph the inequality $x \leq -3$.**

   a. Write the inequality in words. ______________________________

   b. Will the circle at -3 be open or closed? ______________________________

   c. Graph the solution.



2. **Graph the inequality $x < 3$.**

   a. Write the inequality in words. ______________________________

   b. Will the circle at 3 be open or closed? ______________________________

   c. Graph the solution.

Solving Inequalities with Addition and Subtraction

To solve an inequality you can add the same number to or subtract it from each side of the inequality.

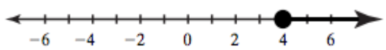
Solve $x + 5 \geq 9$. Graph the solution.

$x + 5 \geq 9$

$x + 5 - 5 \geq 9 - 5$ Subtract 5 from each side.

$x \geq 4$ Simplify.

Graph:



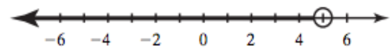
Solve $y - 3 < 2$. Graph the solution.

$y - 3 < 2$

$y - 3 + 3 < 2 + 3$ Add 3 to each side.

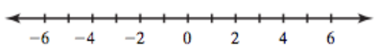
$y < 5$ Simplify.

Graph:

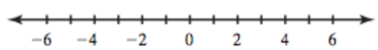


Solve each inequality. Graph the solution.

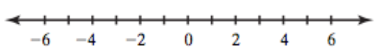
1. $2 + a > 6$ ____________



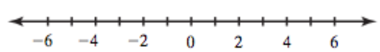
2. $-4 + w \leq 0$ ____________



3. $3 + a \geq 8$ ____________



4. $w + 1 \leq 4$ ____________



Solving Inequalities with Multiplication and Division

To solve an inequality you can multiply or divide each side by the same number. However, if the number is negative, you must also reverse the direction of the inequality sign.

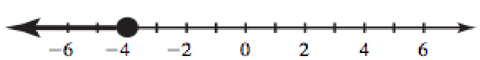
Solve $-4y \geq 16$. Graph the solution.

$-4y \geq 16$

$\frac{-4y}{-4} \leq \frac{16}{-4}$ Divide each side by -4. Reverse the direction of the inequality symbol.

$y \leq -4$ Simplify.

Graph:



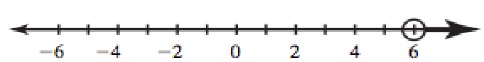
Solve $\frac{w}{3} > 2$. Graph the solution.

$\frac{w}{3} > 2$

$(3)\frac{w}{3} > 2(3)$ Multiply each side by 3.

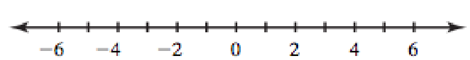
$w > 6$ Simplify.

Graph:

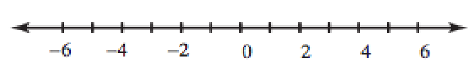


Solve each inequality. Graph the solution.

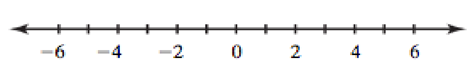
1. $2a > 10$ ____________



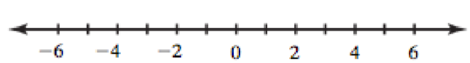
2. $-4w < 16$ ____________



3. $\frac{r}{2} \geq -2$ ____________



4. $\frac{a}{3} < 1$ ____________



Solving Two-Step Inequalities

You can solve a two-step inequality by using inverse operations and the properties of inequality to get the variable alone on one side of the inequality. For many inequalities, first you undo the addition or subtraction, then you undo the multiplication or division—just like when you solve a two-step equation.

Undoing Addition First

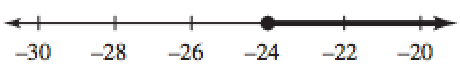
$\frac{x}{2} + 4 \geq -8$

$\frac{x}{2} + 4 - 4 \geq -8 - 4$ ← **Subtract 4 from each side.**

$\frac{x}{2} \geq -12$ ← **Simplify.**

$2 \cdot \frac{x}{2} \geq 2 \cdot (-12)$ ← **Multiply each side by 2.**

$x \geq -24$ ← **Simplify.**



Undoing Subtraction First

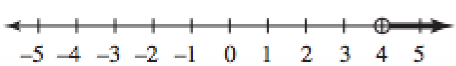
$-2.4y - 7 < -16.6$

$-2.4y - 7 + 7 < -16.6 + 7$ ← **Add 7 to each side.**

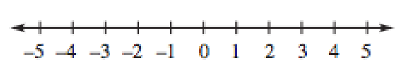
$-2.4y < -9.6$ ← **Simplify.**

$\frac{-2.4y}{-2.4} > \frac{-9.6}{-2.4}$ ← **Divide each side by −2.4.**

$y > 4$ ← **Simplify.**



1. $2a + 4 > 12$ ________________



2. $-3r - 8 > 4$ ________________

Rates

A **rate** is a ratio that compares two quantities measured in different units.

The cost for 10 copies is \$1.50.

The rate is \$1.50/10 copies (\$1.50 per 10 copies).

A **unit rate** is a rate that has a denominator of 1. You can compare using unit rates.

To find the unit rate for 10 copies:

$$\$1.50/10 \text{ copies} = \frac{\$1.50}{10} = \frac{\$1.50 \div 10}{10 \div 10} = \frac{\$.15}{1}$$

The unit rate is \$0.15 per copy. This is also the *unit price.*

COPY CENTER	
Color Copies	
1 copy	\$0.25
10 copies	\$1.50
25 copies	\$2.50
50 copies	\$4.50
100 copies	\$6.00

For the better buy, compare unit rates.

The unit price for 10 copies is \$0.15/copy.

The unit price for 1 copy is \$0.25/copy.

Since $\$0.15 < \$0.25$, the 10-copy price is the better buy.

Find each unit price. Then determine the better buy.

8. juice: 18 oz for \$1.26
8 oz for \$.70

____

9. cloth: 2 yd for \$3.15
6 yd for \$7.78

____

10. socks: 2 pairs for \$3.50
6 pairs for \$9.00

____

11. pecans: 1 lb for \$4.80
2 oz for \$1.00

____

Proportions

A **proportion** is an equation stating that two ratios are equal.

Consider $\frac{2}{10}$ and $\frac{5}{25}$.

$$\frac{2}{10} = \frac{2 \div 2}{10 \div 2} = \frac{1}{5}$$

$$\frac{5}{25} = \frac{5 \div 5}{25 \div 5} = \frac{1}{5}$$

Both ratios are equal to $\frac{1}{5}$, the ratios are proportional.

If two ratios form a proportion, the **cross products** are equal.

$$\frac{100}{2} = \frac{200}{4}$$

$$\frac{100}{2} = \frac{200}{4}$$

$$100 \cdot 4 = 200 \cdot 2$$

$$400 = 400$$

Complete the cross products to determine which pairs of ratios could form a proportion. Then write *yes* or *no*.

1. $\frac{3}{10} \stackrel{?}{=} \frac{6}{20}$

$3 \cdot 20 =$ ______________

$10 \cdot 6 =$ ______________

____

2. $\frac{12}{24} \stackrel{?}{=} \frac{2}{4}$

$12 \cdot 4 =$ ______________

$24 \cdot \square =$ ______________

____

3. $\frac{8}{5} \stackrel{?}{=} \frac{16}{8}$

$8 \cdot \square =$ ______________

$5 \cdot \square =$ ______________

____

Determine if the ratios in each pair are proportional.

4. $\frac{25}{35}, \frac{5}{7}$

____

5. $\frac{15}{3}, \frac{10}{2}$

____

6. $\frac{9}{3}, \frac{12}{4}$

____

Solving Proportions

Solving a proportion means finding a missing part of the proportion. You can use unit rates to solve a proportion. First find the unit rate. Then multiply to solve the proportion.

Shawn filled 8 bags of leaves in 2 hours. At this rate, how many bags would he fill in 6 hours?

① Find a unit rate for the number of bags per hour. Divide by the denominator.

$\frac{8 \text{ bags}}{2 \text{ hours}} = \frac{8 \text{ bags} \boxed{\div 2}}{2 \text{ hours} \boxed{\div 2}} = \frac{4 \text{ bags}}{1 \text{ hour}}$ The unit rate is 4 bags per hour.

② Multiply the unit rate by 6 to find the number of bags he will fill in 6 hours.

Unit rate		Number of hours		Total
↓		↓		↓
4	×	6	=	24

At this rate, Shawn can fill 24 bags in 6 hours.

If two ratios form a proportion, the **cross products** are equal.

Solve. $\frac{5}{15} = \frac{n}{3}$

① Write the cross products. $5 \cdot 3 = 15 \cdot n$

② Simplify. $15 = 15n$

③ Solve the equation. $n = 1$

Solve.

1. The bookstore advertises 5 notebooks for $7.75. At this rate, how much will 7 notebooks cost?

____

Solve each proportion.

6. $\frac{4}{10} = \frac{n}{15}$ ____________ 7. $\frac{4}{200} = \frac{n}{100}$ ____________ 8. $\frac{6}{n} = \frac{5}{10}$ ____________

Is it Proportional?

Compare the ratios to see if there is a proportional relationship.

Hours	2	4	5
Pages	12	24	30

$\frac{12}{2} = \frac{24}{4} = \frac{30}{5}$ ← There is a proportional relationship between pages and hours.

Hours	2	3	4
Pages	10	14	24

$\frac{10}{2} \neq \frac{14}{3} \neq \frac{24}{4}$ ← There is <u>not</u> a proportional relationship between pages and hours.

Determine whether each table represents a proportional relationship. If so, find the constant of proportionality.

1.

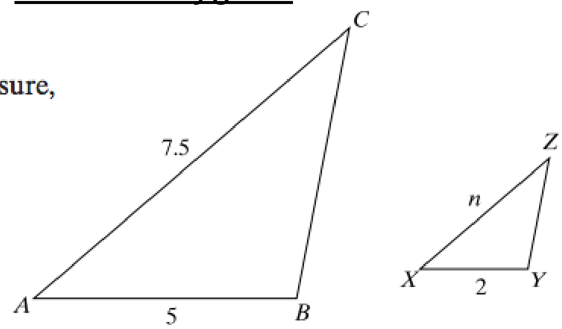
x	4	5	7
y	$10.40	$13.00	$17.50

____

Similar Polygons

Two polygons are **similar** if

- corresponding angles have the same measure, and
- the lengths of corresponding sides are proportional.



$\triangle ABC \sim \triangle XYZ$

You can use proportions to find missing lengths in similar ($\sim$) figures.

① Find corresponding sides.

$\overline{AB}$ corresponds to $\overline{XY}$.
$\overline{AC}$ corresponds to $\overline{XZ}$.
$\overline{BC}$ corresponds to $\overline{YZ}$.

② Write ratios of their lengths in a proportion.

$$\frac{AB}{XY} = \frac{AC}{XZ}$$

③ Substitute the information you know.

$$\frac{5}{2} = \frac{7.5}{n}$$

④ Write cross products. Solve for n.

$$5n = 2 \cdot 7.5$$
$$n = 3$$

The length of $\overline{XZ}$ is 3 units.

The figures are similar. Find the corresponding sides. Then complete the proportion and solve for *n*.

1. $\overline{AB}$ corresponds to ________.

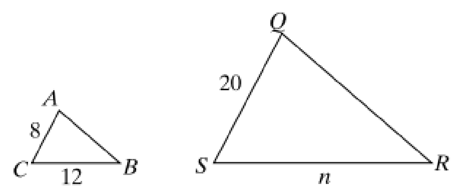
$\overline{BC}$ corresponds to ________.

$\overline{CA}$ corresponds to ________.

2. $\frac{CA}{SQ} = \frac{\square}{RS}$

$\frac{8}{20} = \frac{\square}{\square}$

$n =$ ________



Percents

To write a percent as a fraction, write a fraction with 100 as the denominator.

$45\% = \frac{45}{100}$ ← **Denominator 100**

$= \frac{45 \div 5}{100 \div 5} = \frac{9}{20}$ ← **Simplify.**

$45\% = \frac{9}{20}$

To write a decimal as a percent, multiply by 100.

Write 0.85 as a percent.

$0.85 \cdot 100 = 85$

$0.85 = 85\%$

To write a percent as a decimal, divide by 100.

Write 46% as a decimal.

$46 \div 100 = 0.46$

$46\% = 0.46$

Write each fraction as a percent.

1. $\frac{3}{4}$ ________ **2.** $\frac{12}{25}$ ________ **3.** $\frac{4}{5}$ ________ **4.** $\frac{23}{4}$ ________

Write each percent as a fraction in simplest form.

5. 45% ________ **6.** 60% ________ **7.** 16% ________ **8.** 25% ________

Solving Percent Problems

You can use proportions to solve percent problems. Remember, the percent is compared to 100.

Finding the part:

10% of 40 is _?_.

$\frac{10}{100} = \frac{n}{40}$

$100 \cdot n = 10 \cdot 40$

$n = 4$

10% of 40 is 4.

Finding the whole:

20% of _?_ is 8.

$\frac{20}{100} = \frac{8}{n}$

$20 \cdot n = 100 \cdot 8$

$n = 40$

20% of 40 is 8.

Finding the percent:

_?_ % of 25 is 20.

$\frac{n}{100} = \frac{20}{25}$

$25 \cdot n = 100 \cdot 20$

$n = 80$

80% of 25 is 20.

Complete to solve for *n*.

1. 75% of _?_ is 12.

$\frac{75}{100} = \frac{12}{n}$

$75 \cdot$ ______ $= 100 \cdot$ ______

$n =$ ______

2. 20% of _?_ is 82.

$\frac{20}{100} = \frac{82}{\square}$

$20 \cdot$ ______ $= 100 \cdot$ ______

$n =$ ______

3. 5% of _?_ is 9.

$\frac{5}{100} = \frac{\square}{n}$

______ = ______

$n =$ ______

4. 60 is 5% of n.

$\frac{5}{100} = \frac{\square}{n}$

$5n = 100 \cdot$ ______

$n =$ ______

5. 6% of n is 4.8.

$\frac{6}{\square} = \frac{\square}{n}$

$6n =$ ______ $\cdot 4.8$

$n =$ ______

6. 51 is 170% of n.

$\frac{\square}{100} = \frac{\square}{n}$

______ = ______

$n =$ ______

Percent Applications

Finding Sales Tax

sales tax = percent of tax · purchase price

Find the amount of sales tax on a television that costs $350 with an 8% sales tax.

sales tax = 8% · $350
sales tax = 0.08 · 350
sales tax = 28

The sales tax is $28.

How much does the television cost with sales tax?

$350 + $28 = $378

Find the total cost.

1. $10.00 with a 4% sales tax ______________

2. $8.75 with a 5.25% sales tax ______________

Percent Change

Percent of change is the percent something increases or decreases from its original amount.

	Find the percent of increase from 12 to 18.	Find the percent of decrease from 20 to 12.
① Subtract to find the amount of change.	$18 - 12 = 6$	$20 - 12 = 8$
② Write a proportion. $\frac{\text{change}}{\text{original}} = \frac{\text{percent}}{100}$	$\frac{6}{12} = \frac{n}{100}$	$\frac{8}{20} = \frac{n}{100}$
	$6 \cdot 100 = 12n$	$8 \cdot 100 = 20n$
③ Solve for n.	$n = 50$	$n = 40$
	The percent of increase is 50%.	The percent of decrease is 40%.

State whether the change is an *increase* or *decrease*. Complete to find the percent of change.

1. 40 to 60

$60 - 40 =$ ______

$\frac{\boxed{}}{40} = \frac{n}{100}$

______ $\cdot 100 = 40n$

$n =$ ______________

2. 15 to 9

$15 - 9 =$ ______

$\frac{\boxed{}}{15} = \frac{n}{100}$

______ $\cdot 100 = 15n$

$n =$ ______________

3. 0.4 to 0.9

$0.9 - 0.4 =$ ______

$\frac{\boxed{}}{0.4} = \frac{n}{\boxed{}}$

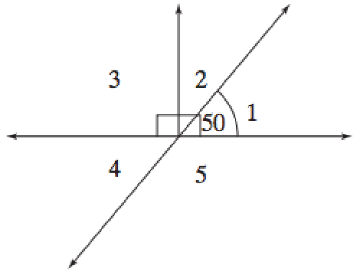
______ $= 0.4n$

$n =$ ______________

Angles

Acute angles have a measurement of less than 90°. In the diagram below, ∠1 and ∠2 are acute angles.

Obtuse angles have a measurement of more than 90°. In the diagram, ∠5 is an obtuse angle.



Vertical angles are formed across from each other where two lines intersect. They always have equal measurements. In the diagram, ∠1 and ∠4 are vertical angles. They both measure 50°.

When the sum of two angles is 90°, the angles are *complementary*. In the diagram, ∠1 and ∠2 are complementary angles. To find the measurement of a complementary angle, subtract from 90°.

$90° - m\angle 1 = m\angle 2$

$90° - 50° = m\angle 2$

$40° = m\angle 2$

$m\angle 2 = 40°$

When the sum of two angles is 180°, the angles are *supplementary*. In the diagram, ∠4 and ∠5 are supplementary angles. To find the measurement of a supplementary angle, subtract from 180°.

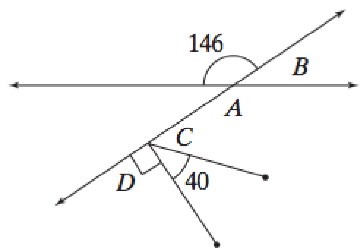
$180° - m\angle 4 = m\angle 5$

$180° - 50° = m\angle 5$

$130° = m\angle 5$

$m\angle 5 = 130°$

Use the diagram to find the measurement of each angle. Then classify each angle as *obtuse, right,* or *acute.*



1. $\angle A$ ________________
2. $\angle B$ ________________
3. $\angle C$ ________________
4. $\angle D$ ________________

Area of Parallelograms

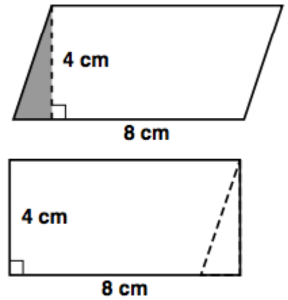
You can use the area of a rectangle to find the area of a parallelogram.

① Draw a perpendicular segment from one vertex to the opposite side to form a triangle.

② Move the triangle to the right side of the parallelogram to form a rectangle.

③ Find the area of the rectangle.
A = length × width = base × height = bh

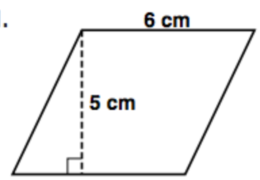
The parallelogram has the same base, height, and area as the rectangle.



$A = bh$
$= 8 \cdot 4$
$= 32\ \text{cm}^2$

Find the area of each figure.

1.



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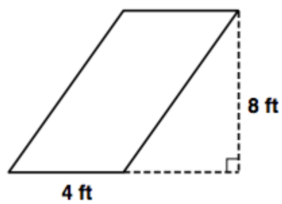

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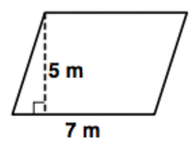
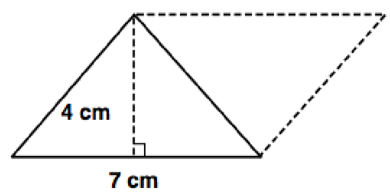


Area of Triangles

You can use the area of a parallelogram to find the area of a triangle. Two identical triangles, together as shown, form a parallelogram. Each triangle has half the area of the parallelogram.



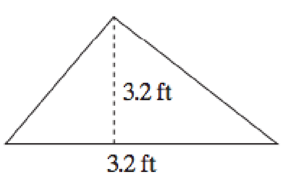
Area of parallelogram: $A = bh$

Area of triangle: $A = \frac{1}{2}bh = \frac{1}{2} \cdot 7 \cdot 4 = 14\ \text{cm}^2$

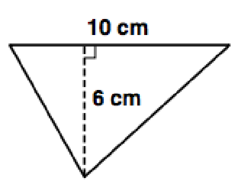
This triangle has an area of 14 cm^2.

Find the area of each triangle.

1.



2.



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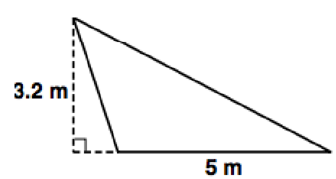
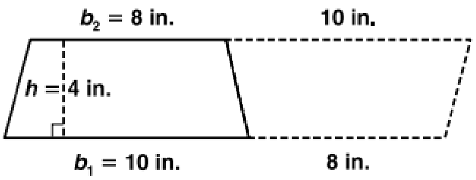


Other Areas

Trapezoid

Two identical trapezoids, together as shown, form a parallelogram. The trapezoid has half the area of the parallelogram.



Area of parallelogram: $A = (b_1 + b_2)h$

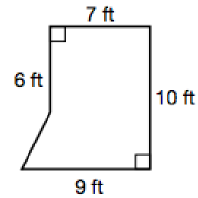
Area of trapezoid: $A = \frac{1}{2}h(b_1 + b_2)$

$= \frac{1}{2}(4)(10 + 8)$

$= 2(18) = 36 \text{ in.}^2$

Irregular Figures

Not all geometric figures are shapes with which you are familiar. Some of them, however, can be divided into familiar shapes.



Find the area of the figure.

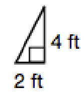
Use the area formulas to find the areas of the triangle and the rectangle.

Area of a triangle $= \frac{1}{2}bh$

$= \frac{1}{2}(2)(4)$

$= \frac{1}{2}(8)$

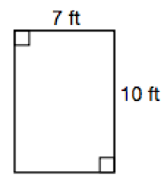
$= 4 \text{ ft}^2$



Area of a rectangle $= bh$

$= (7)(10)$

$= 70 \text{ ft}^2$



Find the total area by adding the area of each figure.

Total area = area of triangle + area of rectangle

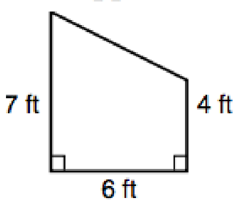
$= 4 + 70$

$= 74$

The total area is 74 ft^2.

Based on appearance, find the area of each figure.

1.



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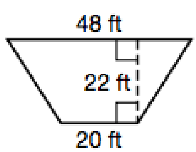
2.



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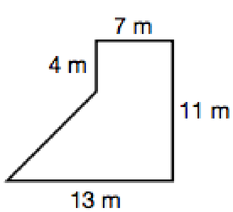
4.



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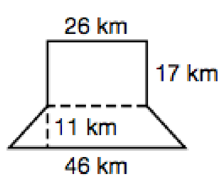
5.



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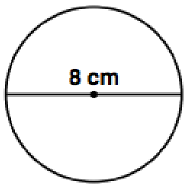
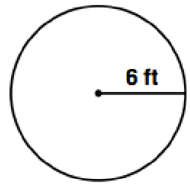
Area and Circumference of Circles

The **circumference** of a circle is the distance around it. To find the circumference of a circle with radius r and diameter d, use either the formula $C = 2\pi r$ or $C = \pi d$. Use 3.14 for π.



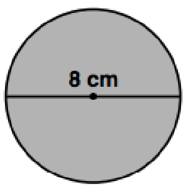
$$
\begin{aligned}
d &= 8 \text{ cm} \\
C &= \pi d \\
&\approx 3.14 \cdot 8 \\
&= 25.12 \text{ cm}
\end{aligned}
$$

To the nearest centimeter, the circumference is 25 cm.



$$
\begin{aligned}
r &= 6 \text{ ft} \\
C &= 2\pi r \\
&\approx 2 \cdot 3.14 \cdot 6 \\
&= 37.68 \text{ ft}
\end{aligned}
$$

To the nearest foot, the circumference is 38 ft.



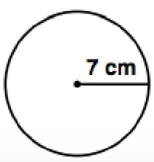
To find the area of a circle, use $A = \pi r^2$.
The diameter of the circle is 8 cm, so the radius is 4 cm.

$$
\begin{aligned}
A &= \pi r^2 \\
&\approx 3.14 \cdot 4 \cdot 4 \\
&= 50.24 \text{ cm}^2
\end{aligned}
$$

To the nearest square centimeter, the area is 50 cm^2.

Find the circumference and area of each circle. Round your answer to the nearest whole unit.

1.



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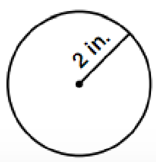

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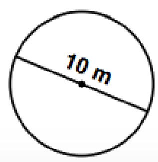
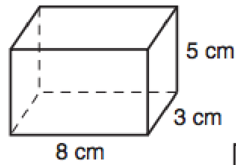
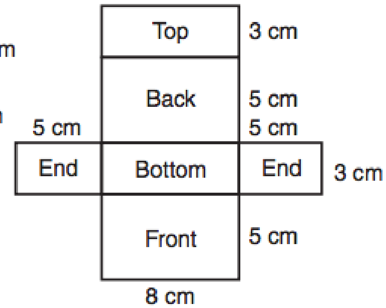


Surface Area and Nets

The **surface area** of a prism is the sum of the areas of its faces. You can use a **net,** or pattern, for the prism to help you find its surface area.

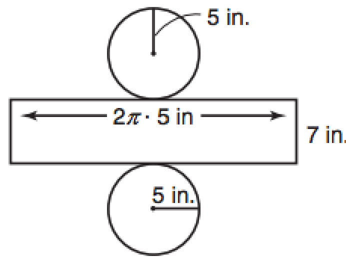


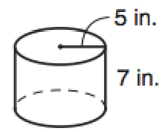


- Add the areas of all the surfaces.

Surface Area
$= \text{front} + \text{back} + \text{top} + \text{bottom} + \text{end} + \text{end}$
$= (8 \cdot 5) + (8 \cdot 5) + (8 \cdot 3) + (8 \cdot 3) + (5 \cdot 3) + (5 \cdot 3)$
$= 40 + 40 + 24 + 24 + 15 + 15$
$= 158 \text{ cm}^2$

- To find the surface area of a cylinder, add the area of the rectangle and the areas of the bases. Use 3.14 for π.

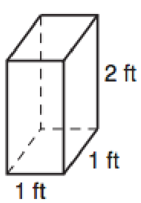


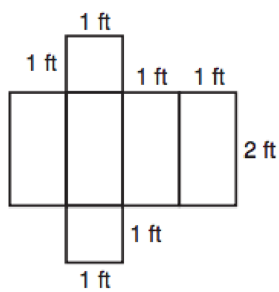


Surface area
$= \text{top} + \text{bottom} + \text{side (rectangle)}$
$= (\pi \cdot 5 \cdot 5) + (\pi \cdot 5 \cdot 5) + (2\pi \cdot 5 \cdot 7)$
$= (25\pi) + (25\pi) + (70\pi)$
$\approx 120 \cdot 3.14 = 376.8 \text{ in.}^2$

Use the net to find the surface area. Round your answers to the nearest whole unit.

1.





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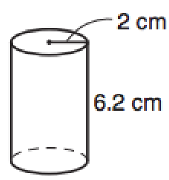

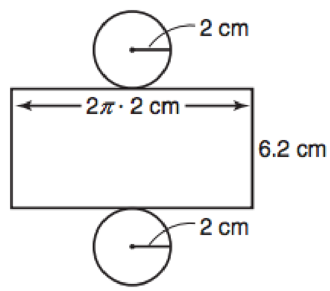

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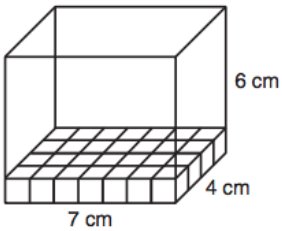
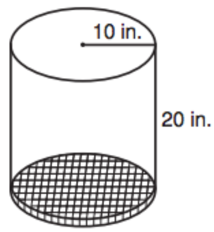
Volume

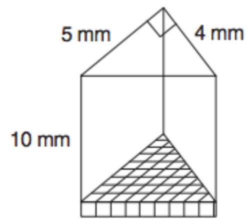
The **volume** of a three-dimensional figure is the number of cubic units needed to fill the space inside the figure. A **cubic unit** is a cube whose edges are 1 unit long. You can find the volume of a prism or a cylinder by finding the *area of the base* (B) and multiplying by the *height* (h). Use 3.14 for π.



$B = lw$
$B = 7 \cdot 4 = 28\text{ cm}^2$
$V = Bh$
$V = 28 \cdot 6 = 168\text{ cm}^3$
The volume is 168 cubic centimeters.



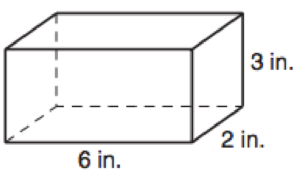
$B = \pi r^2$
$B \approx 3.14 \cdot 10 \cdot 10 = 314\text{ in.}^2$
$V = Bh$
$V \approx 314 \cdot 20 = 6{,}280\text{ in.}^3$
The volume is 6,280 cubic inches.



$B = \frac{1}{2}bh$
$B = \frac{1}{2} \cdot 5 \cdot 4 = 10\text{ mm}^2$
$V = Bh$
$V = 10 \cdot 10 = 100\text{ mm}^3$
The volume is 100 cubic millimeters.

Complete to find the volume to the nearest tenth of a unit.

1.

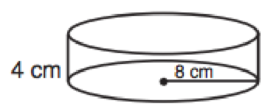


$V = Bh = lwh$

$= \_\_\_\_\_\_ \cdot \_\_\_\_\_\_ \cdot \_\_\_\_\_\_$

$= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$

2.



$V = Bh = \pi r^2 h$

$\approx 3.14 \cdot \_\_\_\_\_\_ \cdot \_\_\_\_\_\_ \cdot \_\_\_\_\_\_$

$= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$

Statistics

To find the average, or mean, of a set of data, you divide the sum of the data values by the number of data values.

You can use information about a sample population to draw inferences about the whole population.

A teacher selected a random sample of 25 middle school students and surveyed them about the number of hours a night they spend on homework. Based on the sample, what is the best estimate of the mean number of hours that middle school students in that school spend on homework every night?

Random Sample of Number of Hours Spent on Homework per Student			
2	4	2	5
4	4	5	5
4	1	1	3
3	3	3	3
2	3	1	2

Step 1 Find the average of the sample data.

$$\text{mean} = \frac{\text{sum of the data values}}{\text{number of data values}}$$

$$= \frac{60}{20}$$

$$= 3$$

Step 2 Use the average of the sample data to draw an inference.

The average time spent on homework in the sample is 3 hours, so the average time spent on homework by all middle school students is likely close to 3.

The table shows the ages of a random sample of 25 customers at a food truck. Use the sample to draw an inference about each measure. Support your answer.

Random Sample of Ages				
14	24	22	38	27
23	19	42	24	39
46	10	11	42	37
34	41	5	19	5
43	50	22	20	43

1. The mean age of customers at the truck ______________
2. The percent of customers under age 21 ______________
3. The percent of customers age 40 and over ______________
4. The median age of customers at the food truck ______________

Theoretical Probability

To find a **theoretical probability,** first list all possible **outcomes.** Then use the formula:

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{total number of possible outcomes}}$$

A letter is selected at random from the letters of the word FLORIDA. What is the probability that the letter is an A?

- There are 7 letters (possible outcomes).
- There is one A, which represents a favorable outcome.

$$P(\text{A}) = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}} = \frac{1}{7}$$

The probability that the letter is an A is $\frac{1}{7}$.

Selecting a letter other than A is called *not* A and is the **complement** of the event A. The sum of the probabilities of an event and its complement equals 1, or 100%.

What is the probability of the event "*not* A"?

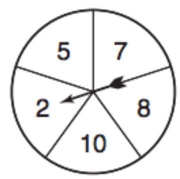
$$P(\text{A}) + P(\textit{not}\ \text{A}) = 1$$

$$\frac{1}{7} + P(\textit{not}\ \text{A}) = 1$$

$$P(\textit{not}\ \text{A}) = 1 - \frac{1}{7} = \frac{6}{7}$$

The probability of the event "*not* A" (selecting F, L, O, R, I, or D) is $\frac{6}{7}$.

Spin the spinner shown once. Find each probability as a fraction, a decimal, and a percent.



1. $P(5)$

   $\frac{\text{number of favorable outcomes}}{\text{total number of outcomes}}$

   $= \frac{\square}{5}$ ________________

2. $P(\text{odd number})$

   $\frac{\text{number of favorable outcomes}}{\text{total number of outcomes}}$

   $= \frac{2}{\square}$ ________________

You select a card at random from a box that contains cards numbered from 1 to 10. Find each probability as a fraction, a decimal, and a percent.

3. $P(\text{even number})$ ________________

4. $P(\text{number less than 4})$ ________________

5. $P(\text{not 5})$ ________________

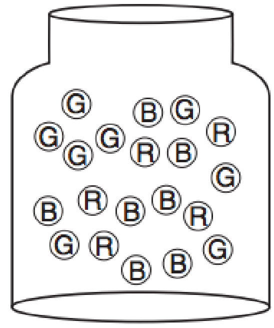
The letters H, A, P, P, I, N, E, S, and S are written on pieces of paper. Select one piece of paper. Find each probability.

6. $P(\text{not vowel})$ ________________

7. $P(\text{not E})$ ________________

Experimental Probability

Probability measures how likely it is that an event will occur. For an **experimental probability,** you collect data through observations or experiments and use the data to state the probability.

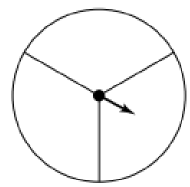


The jar contains red, green, and blue chips. You shake the jar, draw a chip, note its color, and then put it back. You do this 20 times with these results: 7 blue chips, 5 red chips, and 8 green chips. The experimental probability of drawing a green chip is

$$P(\text{green chip}) = \frac{\text{number of times “green chips” occur}}{\text{total number of trials}}$$

$$P(\text{green chip}) = \frac{8}{20} = \frac{2}{5} = 0.4 = 40\%$$

The probability of drawing a green chip is $\frac{2}{5}$, or 0.4, or 40%.



Sometimes a model, or simulation, is used to represent a situation. Then, the simulaton is used to find the experimental probability. For example, spinning this spinner can simulate the probability that 1 of 3 people is chosen for president of the student body.

Use the 20 draws above to complete each exercise.

1. What is the experimental probability of drawing a red chip? Write the probability as a fraction.

$P(\text{red chip}) = \frac{\square}{20} =$ ________________

2. What is the experimental probability of drawing a blue chip? Write the probability as a percent.

$P(\text{blue chip}) = \frac{\square}{\square} =$ ________________

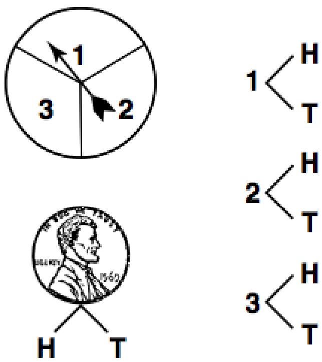
Suppose you have a bag with 30 chips: 12 red, 8 white, and 10 blue. You shake the jar, draw a chip, note its color, and then put it back. You do this 30 times with these results: 10 blue chips, 12 red chips, and 8 white chips. Write each probability as fraction in simplest form.

3. $P(\text{red})$ ________________ **4.** $P(\text{white})$ ________________ **5.** $P(\text{blue})$ ________________

<u>Sample Space</u>

The set of all possible outcomes of an experiment is called the **sample space.**

You can use a *tree diagram* or a table to show the sample space for an experiment. The tree diagram below shows the sample space for spinning the spinner and tossing a coin.



There are 6 possible outcomes: 1H, 1T, 2H, 2T, 3H, 3T. What is the probability of spinning a 3 and tossing heads? There is one favorable outcome (3H) out of 6 possible outcomes. The probabilty is $\frac{1}{6}$.

You can use the *counting principle* to find the number of possible outcomes: If there are m ways of making one choice and n ways of making a second choice, then there are $m \times n$ ways of making the first choice followed by the second.

Evelyn and Kara are planning to go skating or to a movie. Afterward they want to go out for pizza, tacos, or cheeseburgers. How many possible choices do they have?

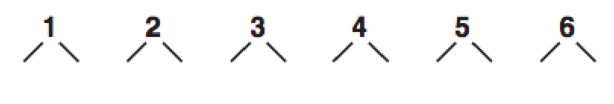
- There are *two choices* for an activity and *three choices* for food.
- First choices × Second choices

$$2 \quad \times \quad 3 \quad = 6$$

There are 6 possible choices.

Complete the tree diagram to show the sample space.

1. Roll a number cube and toss a coin. What is the probability of getting (4, Heads)?



Number of outcomes ____________________

P(4, heads) = ___________________________

Use the counting principle to find the number of possible outcomes.

2. 4 kinds of yogurt and 8 toppings

____

3. 6 shirts and 9 pairs of slacks

____

Dependent and Independent Outcomes

If you toss a coin and roll a number cube, the events are **independent.** The outcome of one event does not affect the outcome of the second event.

Find the probability of tossing a heads (H) and rolling an even number (E).

Find $P(\text{H and E})$. H and E are independent.

① Find $P(\text{H})$:

$$P(\text{H}) = \frac{1 \text{ heads}}{2 \text{ sides}} = \frac{1}{2}$$

② Find $P(\text{E})$:

$$P(\text{E}) = \frac{3 \text{ evens}}{6 \text{ faces}} = \frac{1}{2}$$

③ $P(\text{H and E}) = P(\text{H}) \times P(\text{E}) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

If the outcome of the first event affects the outcome of the second event, the events are **dependent.**

A bag contains 3 blue and 3 red marbles. Draw a marble, then draw a second marble without replacing the first marble. Find the probability of drawing 2 blue marbles.

① Find $P(\text{blue})$.

$$P(\text{blue}) = \frac{3 \text{ blue}}{6 \text{ marbles}} = \frac{1}{2}$$

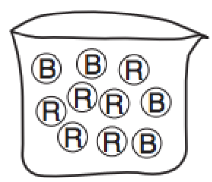
② Find $P(\text{blue after blue})$.

$$P(\text{blue after blue}) = \frac{2 \text{ blue}}{5 \text{ marbles}} = \frac{2}{5}$$

③ Find $P(\text{blue, then blue})$

$$P(\text{blue, then blue}) = P(\text{blue}) \times P(\text{blue after blue}) = \frac{1}{2} \times \frac{2}{5} = \frac{1}{5}$$

In Exercises 1–3, you draw a marble at random from the bag of marbles shown. Then, you replace it and draw again. Find each probability.



1. $P(\text{blue, then red})$ ________
2. $P(\text{2 reds})$ ________
3. $P(\text{2 blues})$ ________

Next, you draw two marbles randomly *without* replacing the first marble. Find each probability.

4. $P(\text{blue, then red})$ ________
5. $P(\text{2 reds})$ ________
6. $P(\text{2 blues})$ ________